

3. A uniform rectangular lamina $ABCD$, where $AB = a$ and $BC = 2a$, has mass $2m$. The lamina is free to rotate about its edge AB , which is fixed and vertical. The lamina is at rest when it is struck at C by a particle P of mass m . The particle P is moving horizontally with speed U in a direction which is perpendicular to the lamina. The coefficient of restitution between P and the lamina is 0.5

Find the angular speed of the lamina immediately after the impact.

(8)



5. A particle moves in a plane so that its position vector \mathbf{r} metres at time t seconds satisfies the differential equation

$$\frac{d\mathbf{r}}{dt} + (\tan t)\mathbf{r} = (\cos^2 t)\mathbf{i} - (3\cos t)\mathbf{j}, \quad 0 \leq t < \frac{\pi}{2}$$

When $t = 0$, the particle is at the point with position vector $4\mathbf{j}$ m.

Find \mathbf{r} in terms of t .

(8)



Question 5 continued

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6. Three forces \mathbf{F}_1 , \mathbf{F}_2 and \mathbf{F}_3 act on a rigid body at the points with position vectors, \mathbf{r}_1 , \mathbf{r}_2 and \mathbf{r}_3 respectively, where

$$\mathbf{F}_1 = (2\mathbf{i} - \mathbf{j} + \mathbf{k}) \text{ N} \quad \mathbf{F}_2 = (3\mathbf{i} + \mathbf{j} - 2\mathbf{k}) \text{ N} \quad \mathbf{F}_3 = (-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) \text{ N}$$

$$\mathbf{r}_1 = (\mathbf{i} - \mathbf{k}) \text{ m} \quad \mathbf{r}_2 = (2\mathbf{i} - \mathbf{j} + \mathbf{k}) \text{ m} \quad \mathbf{r}_3 = (\mathbf{i} + \mathbf{j} - \mathbf{k}) \text{ m}$$

The system of the three forces is equivalent to a single force \mathbf{R} acting at the point with position vector $(3\mathbf{i} - \mathbf{j} + \mathbf{k}) \text{ m}$, together with a couple of moment \mathbf{G} .

(a) Find \mathbf{R} . (2)

(b) Find \mathbf{G} . (9)



8.

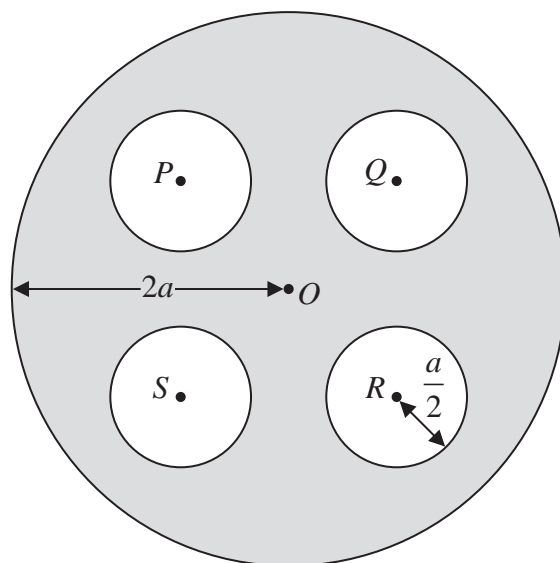


Figure 1

A uniform circular disc of radius $2a$ has centre O . The points P , Q , R and S on the disc are the vertices of a square with centre O and $OP = a$. Four circular holes, each of radius $\frac{a}{2}$, and with centres P , Q , R and S , are drilled in the disc to produce the lamina L , shown shaded in Figure 1. The mass of L is M .

- (a) Show that the moment of inertia of L about an axis through O , and perpendicular to the plane of L , is $\frac{55Ma^2}{24}$ (8)

The lamina L is free to rotate in a vertical plane about a fixed smooth horizontal axis which is perpendicular to L and which passes through a point A on the circumference of L . At time t , AO makes an angle θ with the downward vertical through A .

- (b) Show that $\frac{d^2\theta}{dt^2} = -\frac{48g}{151a}\sin\theta$ (4)

- (c) Hence find the period of small oscillations of L about its position of stable equilibrium. (2)

The magnitude of the component, in a direction perpendicular to AO , of the force exerted on L by the axis is X .

- (d) Find X in terms of M , g and θ . (4)



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Question 8 continued

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Q8

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(Total 18 marks)

TOTAL FOR PAPER: 75 MARKS

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